

Measuring the galaxy power spectrum and scale-scale correlations with multiresolution-decomposed covariance – I. method

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ABSTRACT

We present a method of measuring galaxy power spectrum based on the multiresolution analysis of the discrete wavelet transformation (DWT). Besides the technical advantages of the computational feasibility for data sets with large volume and complex geometry, the DWT scale-by-scale decomposition provides a physical insight into the covariance matrix of the cosmic mass field. Since the DWT representation has strong capability of suppressing the off-diagonal components of the covariance for selfsimilar clustering, the DWT covariance for all popular models of the cold dark matter cosmogony generally is diagonal, or $j(\text{scale})$ -diagonal in the scale range, in which the second or higher order scale-scale correlations are weak. In this range, the DWT covariance gives a lossless estimation of the power spectrum, which is equal to the corresponding Fourier power spectrum banded with a logarithmical scaling. This DWT estimator is optimized in the sense that the spatial resolution is adaptive automatically to the perturbation wavelength to be studied. In the scale range, in which the scale-scale correlation is significant, the accuracy of a power spectrum detection depends on the scale-scale or band-band correlations. In this case, for a precision measurements of the power spectrum, or a precision confrontation of the observed power spectrum with models, a measurement of the scale-scale or band-band correlations is needed. We show that the DWT covariance can be employed to measuring both the band-power spectrum and second order scale-scale correlation.

We also present the DWT algorithm of the binning and Poisson sampling with real observational data. We show that the so-called alias effect appeared in usual binning schemes can exactly be eliminated by the DWT binning. Since Poisson process possesses diagonal covariance in the DWT representation,

the Poisson sampling and selection effects on the power spectrum and second order scale-scale correlation detection are suppressed into minimum. Moreover, the effect of the non-Gaussian features of the Poisson sampling can also be calculated in this frame. The DWT method is open, i.e. one can add further DWT algorithms on the basic decomposition in order to estimate other effects on the power spectrum detection, such as non-Gaussian correlations and bias models.

Subject headings: cosmology: theory - large-scale structure of the universe

1. Introduction

Measuring the galaxy power spectrum has been and is being a central subject of the large scale structure study. Although the power spectrum is only a second order statistical measure of the deviations of a random field, $\delta(\mathbf{x})$, of mass density from homogeneity, it directly reflects the physical scales of the processes that affect structure formation. Mathematically, the positive definiteness of the power spectrum is useful for constraining the parameter space in comparing predictions with data. Since the ongoing and upcoming redshift surveys of galaxies will provide data of galaxy distribution with highly improved quality and a larger quantity, it also requests to develop the methods of measuring the power spectrum more precise and computationally efficient.

Different methods of the power spectrum measurements adopt different representations, or decomposition of the covariance $Cov = \langle \delta(\mathbf{x})\delta(\mathbf{x}') \rangle$, where $\langle \dots \rangle$ stands for an ensemble average. For a representation given by a set of basis functions $\psi_i(\mathbf{x})$ (sometimes referred as weight function), the random field is described by the variables

$$X_i = \int \delta(\mathbf{x})\psi_i(\mathbf{x})d\mathbf{x}, \quad (1)$$

and the covariance is given by $Cov_{ij} = \langle X_i X_j \rangle$. If the covariance in this representation is exactly or approximately diagonalized, the diagonal elements $\langle |X_i|^2 \rangle$ would be a fair estimate of the power spectrum, or band-power spectrum. Thus, measuring power spectrum mathematically is almost a synonym of diagonalizing the covariance of the density field $\delta(\mathbf{x})$, or calculating the eigenvalues of the covariance matrix.

Traditionally, the Fourier decomposition, and then, the Fourier power spectrum are the popular tool to analyze a cosmic density field, because the Fourier transform retains the translation invariance of a homogeneous and isotropic universe. However, the observed sample given by redshift surveys are not translation invariant due to the selection effect and

irregular geometry of the surveys. To effectively compare the predicted power spectrum with the observed galaxy distributions, the basis functions of the decomposition should be chosen to incorporate with the selection effect, sampling, and complex geometry of the data. As a result, various decompositions for measuring the galaxy power spectrum have been proposed (Tegmark, et al. 1998 and reference therein). An ideal estimator of the power spectrum should match the following conditions

- X_i s are independent from each other, i.e. the data is decomposed into mutually exclusive chunks;
- X_i s retains all the information of the original data, i.e. the decomposed chunks are collectively exhaustive;
- It is computationally feasible;
- It allows us to take account of the systematic effects, such as redshift distortion, evolution, morphology-dependence, galactic extinction etc.

These ideal estimators are believed to be information lossless, i.e. retaining all information of the power spectrum in the original data.

We will study, in this paper, the estimator based on the multiscale decomposition, i.e. the discrete wavelet transform (DWT) representation. The DWT power spectrum estimator has been applied to measure the power spectrum from samples of the Ly- α forests of QSO's absorption spectra (Pando & Fang 1998a.) The result has demonstrated that the DWT power spectrum estimator can match the conditions listed above, especially it is very helpful to overcome the difficulties of complex geometry and sampling. Within the framework of DWT, this paper will present a general working scheme for extracting the statistical characters from the observational data, in which the selection effect, sampling and binning are addressed.

It has been recognized recently that the non-Gaussian behavior of X_i is substantial for a precise measurement of the power spectrum. The accuracy of a power spectrum estimation is significantly affected by the so-called power spectrum correlations induced by non-linear clustering (Meiksin & White, 1998, Scoccimarro, Zaldarriaga & Hui 1999). The power spectrum correlation is also found to be essential for recovering the initial power spectrum by a Gaussianization of observed distribution (Weinberg 1992, Narayanan & Weinberg 1998, Feng & Fang 1999). Thus, beyond the conditions mentioned above for an ideal power spectrum estimator, one should add one more requirement that the power spectrum correlation caused by the non-linear clustering and Poisson sampling are calculable. We will show that the power spectrum correlations, or the scale-scale correlations, can be calculated in the DWT analysis.

Moreover, for popular models of the cold dark matter cosmogony, including the standard cold dark matter models (SCDM), open CDM model (OCDM), and flat CDM (LCDM), the scale-scale correlations have been found to be negligible on large scales, and the non-local scale-scale correlations are also negligible even on small scales (Fang, Deng & Fang 2000). That is, the effect of the power spectrum correlations is largely suppressed in the DWT representation. We will show how to take the advantage of this suppression for a scale-by-scale approach of measuring the power spectrum.

The paper will be organized as follows. §2 gives a brief description of the DWT decomposition of the covariance of density random field. The physical meaning and mathematical properties of the j diagonal and j off-diagonal components of the covariance will also be discussed. In §3, an optimized band power spectrum estimator based on the DWT j diagonal covariance is proposed. In addition, the scale-scale correlation extracting from the j off-diagonal components of the covariance is investigated. This correlation gives the scale range in which the power spectrum obtained by the j diagonalization

are information lossless. We then present the algorithm for estimating the DWT band power spectrum from observed galaxy catalog. It includes the DWT binning (§4), and the DWT technique of dealing with Poisson sampling and selection (§5). The discussions and conclusions are given in §6. A brief introduction of the DWT analysis is given in Appendix.

2. Covariance of density fluctuations in the DWT representation

2.1. DWT decomposition of density fields

For the sake of simplicity, we analyze a 1-D density distribution $\rho(x)$ in the range $0 < x < L$, which is assumed to be a stationary random field. The density contrast is defined by $\delta(x) = (\rho(x) - \bar{\rho})/\bar{\rho}$, where $\bar{\rho} = \langle \rho(x) \rangle$, and $\langle \dots \rangle$ stands for ensemble average. It would be straightforward to extend the most results to 2-D and 3-D. Some specific problems related with higher dimension extension will be discussed in §6. In addition, the redshift distortion will not be taken into account in this paper.

To ensure a multiscale decomposition of $\delta(x)$ to be information-lossless, the natural working scheme is to adopt discrete wavelet transformation (DWT) within the framework of multiresolution analysis (MRA). The mathematical construction of MRA theory is briefly sketched in Appendix A.

Let $\delta^P(x)$ be the periodic extension of $\delta(x)$, i.e., $\delta^P(x) = \delta(x - [x/L] \cdot L)$, where $[\eta]$ denotes integer part of η . From eq.(A36), the density contrast $\delta^P(x)$ can be decomposed in term of orthonormal wavelet basis

$$\delta^P(x) = \sum_{j=0}^{\infty} \sum_{l=-\infty}^{+\infty} \tilde{\epsilon}_{j,l} \psi_{j,l}(x), \quad (2)$$

The wavelet function coefficient (WFC), $\tilde{\epsilon}_{j,l}$, is given by the inner product of

$$\tilde{\epsilon}_{j,l} = \langle \psi_{j,l} | \delta \rangle \equiv \int_{-\infty}^{\infty} \delta^P(x) \psi_{j,l}(x) dx. \quad (3)$$

which describes the density fluctuation on scale $L/2^j$ at position $lL/2^j$. The WFCs are the variables of the random field in the DWT representation. The original distributions can be exactly and unredundantly reconstructed from these decomposed variables.

By using the periodized wavelet function defined by

$$\psi_{j,l}^P(x) = \left(\frac{2^j}{L}\right)^{1/2} \sum_{n=-\infty}^{\infty} \psi[2^j(\frac{x}{L} + n) - l]. \quad (4)$$

where ψ is the basic wavelet function [eq.(A21)], eq.(1) becomes

$$\delta^P(x) = \sum_{j=0}^{\infty} \sum_{l=-0}^{2^j-1} \tilde{\epsilon}_{j,l} \psi_{j,l}^P(x), \quad (5)$$

The WFC can then be computed by

$$\tilde{\epsilon}_{j,l}^P = \int_0^L \delta^P(x) \psi_{j,l}^P(x) dx \quad (6)$$

We will always use the periodized functions below, and drop the superscript P .

Furthermore, $\psi_{j,l}(x)$ is admissible [eq.(A27)], which implies that $\psi_{j,l}(x)$ has zero mean if it is integrable,

$$\int \psi_{j,l}(x) dx = 0. \quad (7)$$

It then follows from eq.(2) that

$$\langle \tilde{\epsilon}_{j,l} \rangle = 0 \quad (8)$$

The Fourier decomposition of the field $\delta(x)$ is given by

$$\delta(x) = \sum_{n=-\infty}^{\infty} \delta_n e^{i2\pi nx/L}, \quad (9)$$

where n is an integer, and the Fourier coefficients, δ_n , is

$$\delta_n = \langle n | \delta \rangle \equiv \frac{1}{L} \int_0^L \delta(x) e^{-i2\pi nx/L} dx, \quad (10)$$

Since both the bases of the Fourier transform and the DWT are orthogonal and complete in the space of 1-D functions with period length L , we have

$$\sum_{j=0}^{\infty} \sum_{l=0}^{2^j-1} \langle n | \psi_{j,l} \rangle \langle \psi_{j,l} | n' \rangle = \delta_{n,n'}^K \quad (11)$$

where $\delta_{n,n'}^K$ is the Kronecker Delta function, and $\langle n | \psi_{j,l} \rangle$ the Fourier transform of the wavelet $\psi_{j,l}$ given by

$$\hat{\psi}_{j,l}(n) \equiv \langle n | \psi_{j,l} \rangle = \int_0^L \psi_{j,l}(x) e^{-i2\pi nx/L} dx. \quad (12)$$

Considering the wavelet $\psi_{j,l}(x)$ is related to the basic wavelet $\psi(\eta)$ by eq.(A11), eq.(12) can be rewritten as

$$\hat{\psi}_{j,l}(n) = \left(\frac{2^j}{L} \right)^{-1/2} \hat{\psi}(n/2^j) e^{-i2\pi nl/2^j}, \quad (13)$$

where $\hat{\psi}(n)$ is the Fourier transform of the basic wavelet

$$\hat{\psi}(n) = \int_0^L \psi(\eta) e^{-i2\pi n\eta} d\eta. \quad (14)$$

Substituting expansion (9) into eq.(6) yields

$$\tilde{\epsilon}_{j,l} = \sum_{n=-\infty}^{\infty} \delta_n \int_0^L e^{i2\pi nx/L} \psi_{j,l}(x) dx = \sum_{n=-\infty}^{\infty} \delta_n \hat{\psi}_{j,l}(-n). \quad (15)$$

Similarly, inserting expansion (5) into eq.(10) we have

$$\begin{aligned} \delta_n &= \frac{1}{L} \sum_{j=0}^{\infty} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l} \hat{\psi}_{j,l}(n) \\ &= \sum_{j=0}^{\infty} \sum_{l=0}^{2^j-1} \left(\frac{1}{2^j L} \right)^{1/2} \tilde{\epsilon}_{j,l} e^{-i2\pi nl/2^j} \hat{\psi}(n/2^j), \quad n \neq 0. \end{aligned} \quad (16)$$

Equations (15) and (16) show that both the Fourier variables δ_n and the DWT variables $\tilde{\epsilon}_{j,l}$ are complete.

However, the statistical properties of the Fourier mode n and the DWT mode (j, l) are quite different. For a non-Gaussian field consisting of randomly homogeneously

distributed clumps with a non-Gaussian probability distribution function(PDF), the one-point distributions of the real and imaginary components of the Fourier modes could be still Gaussian. That is because the Fourier modes are subject to the central limit theorem of random fields (Adler 1981). Even though the non-Gaussian clumps are correlated, the central limit theorem still holds if the two-point correlation function of the clumps approaches zero fast sufficiently (Fan & Bardeen, 1995.) Thus, the non-Gaussian information could be lost in the Fourier representation if the phases of the Fourier coefficients are missing.

On the other hand, the DWT basis doesn't suffer from the central limit theorem. A key condition necessary for the central limit theorem to hold is that the modulus of the decomposition basis are less than C/\sqrt{L} , where L is the size of the sample and C is a constant (Ivanov & Leonine 1989). The Fourier basis obviously satisfy this condition because of $(1/\sqrt{L})|\sin 2\pi nx/L| < C/\sqrt{L}$, where C is independent of x and n . While the DWT basis is compactly supported (Appendix A), and its modulus does not satisfy the condition $< C/\sqrt{L}$. Consequently, for the non-Gaussian fields, the one-point distributions of the Fourier variables $|\delta_n|$ could be Gaussian, while for the DWT variable $\tilde{\epsilon}_{j,l}$, the one-point distributions show non-Gaussian (Pando & Fang 1998b.)

2.2. The WFC covariance and DWT power spectrum

In the DWT representation, the covariance $\langle \delta(x)\delta(x') \rangle$ is expressed by a matrix $\langle \tilde{\epsilon}_{j,l}\tilde{\epsilon}_{j',l'} \rangle$ with subscripts $(j,l); (j',l')$. The elements of $j = j', l = l'$ will be called diagonals, while $j = j'$ called j diagonals, and $j \neq j'$ the j off-diagonals.

The Parseval's theorem for the DWT decomposition is (Fang & Thews 1998)

$$\frac{1}{L} \int_0^L |\delta(x)|^2 dx = \sum_{j=0}^{\infty} \frac{1}{L} \sum_{l=0}^{2^j-1} |\tilde{\epsilon}_{j,l}|^2, \quad (17)$$

which implies that the power of perturbations can be divided into modes, (j, l) . $|\tilde{\epsilon}_{j,l}|^2$ describes the power of the mode (j, l) . One can then define the DWT power spectrum by the diagonals of the covariance matrix, i.e.³

$$P_{j,l} = \langle \tilde{\epsilon}_{j,l}^2 \rangle. \quad (18)$$

Since the random variables $\tilde{\epsilon}_{j,l}$ are complete, one can define a Gaussian field $\delta(x)$ by requiring that all the variables $\tilde{\epsilon}_{j,l}$ are distributed as a Gaussian process with the covariance

$$\langle \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j',l'} \rangle = P_{j,l} \delta_{j,j'} \delta_{l,l'}, \quad (19)$$

and the zero ensemble average of all higher order cumulants of $\tilde{\epsilon}_{j,l}$. Thus, a Gaussian field is completely described by its DWT power spectrum $P_{j,l}$. For a homogeneous Gaussian field, the DWT power spectrum $P_{j,l}$ is l -independent, i.e. $P_{j,l} = P_j$.

Using eqs.(15) and (16), the covariance in the Fourier and DWT representations can be converted from one form to another by

$$\langle \hat{\delta}_n \hat{\delta}_{n'}^\dagger \rangle = \sum_{j,j'=0}^{+\infty} \sum_{l=0}^{2^j-1} \sum_{l'=0}^{2^{j'}-1} \langle \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j',l'} \rangle \hat{\psi}_{j,l}(n) \hat{\psi}_{j',l'}^\dagger(n') \quad (20)$$

and conversely

$$\langle \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j',l'} \rangle = \sum_{n,n'=-\infty}^{+\infty} \langle \hat{\delta}_n \hat{\delta}_{n'}^\dagger \rangle \hat{\psi}_{j',l'}(n') \hat{\psi}_{j,l}^\dagger(n). \quad (21)$$

Therefore, for a homogeneous Gaussian field given by the DWT power spectrum P_j , eq. (20) implies

$$\langle \delta_n \delta_{n'}^\dagger \rangle = P(n) \delta_{n,n'}, \quad (22)$$

where

$$P(n) = \sum_{j=0}^{\infty} P_j \left| \hat{\psi} \left(\frac{n}{2^j} \right) \right|^2. \quad (23)$$

³The DWT power spectrum, or called scalogram, has been extensively applied in signal analysis (e.g. Mallat 1999.)

In the derivation of eqs.(22), we used

$$\sum_{l=0}^{2^j-1} e^{-i2\pi(n-n')l/2^j} = \delta_{n,n'}. \quad (24)$$

Eq.(22) shows that for a homogeneous Gaussian P_j , the Fourier power spectrum $P(n)$ is uniquely determined by the DWT power spectrum P_j .

However, the reversed relation doesn't exist, i.e. one *cannot* show that the DWT covariance is given by eq.(19) with $P_{j,l} = P_j$ if the Fourier covariance is given by eq.(22). This indicates that the Fourier and WFC covariance are not equivalent. For instance, fields consisting of homogeneously distributed non-Gaussian clumps generally do not satisfy eq.(19) with a l -independent $P_{j,l}$, but do so for eq.(22). That is, eq.(19) with a l -independent $P_{j,l}$ places stronger constraints on the random field than eq.(22), and therefore, eq.(22) will hold when eq.(19) with a l -independent $P_{j,l}$ holds, but not generally true for the converse.

2.3. j off-diagonals of the WFC covariance

We now identify the physical meaning of the j off-diagonal components of the WFC covariance.

When the “fair sample hypothesis” (Peebles 1980) holds, or equivalently, the random field is ergodic, the 2^j WFCs $\tilde{\epsilon}_{j,l}$, $l = 0 \dots 2^j - 1$, for a given j can be taken as 2^j independent measurements, because they are measured by projecting onto the mutually orthogonal basis $\psi_{j,l}(x)$. Accordingly, the 2^j WFCs form a statistical ensemble on the scale j . This ensemble represents actually the one-point distribution of the fluctuations of the DWT modes at a given scale j . The average over l is thus a fair estimation of the ensemble average.

For a Gaussian field, these one-point distributions are Gaussian. However, even if the one-point distributions for all j are Gaussian, the density field $\delta(x)$ could still be non-Gaussian. That is simply due to the statistical properties of the WFCs $\tilde{\epsilon}_{j,l}$ for indices

j and l are independent. It is easy to construct a density field $\delta(x)$ for which the WFCs $\tilde{\epsilon}_{j,l}$ are Poisson or Gaussian in its one-point distribution with respect to l , while highly non-Gaussian in terms of j (Greiner, Lipa & Carruthers 1995). A simple example is demonstrated as follows. Suppose the one-point distribution of the 2^j WFCs, $\tilde{\epsilon}_{j,l}$, on a scale j , is Gaussian. If the WFCs on the scale $j+1$ is incorporated with those on the scale j , e.g.,

$$\begin{aligned}\tilde{\epsilon}_{j+1,2l} &= a\tilde{\epsilon}_{j,l}, \\ \tilde{\epsilon}_{j+1,2l+1} &= b\tilde{\epsilon}_{j,l},\end{aligned}\tag{25}$$

where a and b are arbitrary constants, the one-point distribution of the 2^{j+1} WFCs $\tilde{\epsilon}_{j+1,l}$ is also Gaussian. However, the coherent structure given by eq.(25) leads to a strong correlation between $\tilde{\epsilon}_{j+1,l}$ and $\tilde{\epsilon}_{j,l}$, i.e. the scale $j+1$ fluctuations are always proportional to those on the scale j at the same position. This is a local scale-scale correlation. One can also design non-local scale-scale correlation by

$$\begin{aligned}\tilde{\epsilon}_{j+1,2l} &= a\tilde{\epsilon}_{j,l+\Delta l}, \\ \tilde{\epsilon}_{j+1,2l+1} &= b\tilde{\epsilon}_{j,l+\Delta l},\end{aligned}\tag{26}$$

where $\Delta l = 1, 2, \dots$. Eq.(26) leads to a strong correlation between the fluctuations on scales $j+1$ and j , but at two places with distance Δl .

Hence, in terms of the DWT representation, a homogeneous Gaussian field requires that (1) the one-point distributions of the WFCs with respect to l are Gaussian, and (2) the distributions of WFCs with different j 's are uncorrelated, such as

$$\langle \tilde{\epsilon}_{j+1,l} \tilde{\epsilon}_{j,l'} \rangle = 0.\tag{27}$$

Correspondingly, in the Fourier representation, a Gaussian field also has two requirements (1) the one-point distributions of the amplitudes of the Fourier mode $|\delta_n|$ are Gaussian; (2) the phases of δ_n are random. Therefore, eq.(27) is the DWT counterpart of

the Fourier random phase. However, it is difficult, or practically impossible, to capture the phase information of each Fourier modes. The local scale-scale correlation is overlooked with the Fourier covariance.

In summary, the j off-diagonals of the WFC covariance provide the information of the scale-scale correlation. This non-Gaussian feature arises from mode-mode coupling of gravitational clustering, and cannot be measured by the higher order cumulants of the one-point distribution for a given scale j , rather, the cross correlation between the different scales. The covariance of a system without scale-scale correlation will be j -diagonal, i.e.

$$\langle \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j',l'} \rangle = \langle \tilde{\epsilon}_{j,l} \rangle \langle \tilde{\epsilon}_{j',l'} \rangle = 0, \quad j \neq j', \quad (28)$$

where eq.(8) has been used at the last step.

3. Statistical information extracting from the WFC covariance

3.1. j -diagonalization of the WFC covariance

It has been known that the DWT is powerful for data compression. For very wide types of stochastic clustering processes, the off-diagonal components of the covariance are strongly suppressed in the DWT representation. This suppression is especially efficient for selfsimilar clustering. For instance, one can show analytically that the covariance in the DWT representation is exactly diagonal for some popular hierarchical models of structure formations, such as the block model and its variants (Meneveau & Sreenivasan 1987, Cole & Kaiser 1988). In this respect, the DWT basis represents the adequate normal coordinates. In other words, the DWT analysis can be understood as a Proper Orthonormal Decomposition (POD), or a Karhunen-Loève transformation (e.g. Aubry et al. 1988), in regard to the second order correlations of these stochastic clustering processes.

For more realistic models and observed samples, the WFC covariance is not fully diagonal, but mostly j -diagonal. In fact, this character has been evident from the measurement of the fourth order scale-scale correlation in the observational samples such as the Ly α forest lines (Pando et al. 1998), the transmitted flux of QSO absorption spectrum (Feng & Fang 1999) and the APM bright galaxy catalog (Feng, Deng & Fang 2000). A common conclusion is that the scale-scale correlations are very weak, and negligible on large scales, i.e. $\langle \tilde{\epsilon}_{j,l}^2 \tilde{\epsilon}_{j',l'}^2 \rangle = \langle \tilde{\epsilon}_{j,l}^2 \rangle \langle \tilde{\epsilon}_{j',l'}^2 \rangle$ for $j \neq j'$ and $j, j' \leq J_{ss}$, where J_{ss} denotes the scale above which the scale-scale correlation is not significant. It is also true for the mass distributions and 2-D and 3-D mock catalog of galaxies in the CDM family of models (Feng, Deng & Fang 2000). This result indicates $\langle \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j',l'} \rangle = \langle \tilde{\epsilon}_{j,l} \rangle \langle \tilde{\epsilon}_{j',l'} \rangle = 0$ for $j \neq j'$ and $j, j' \leq J_{ss}$. Of course, the typical scale J_{ss} relies on the models or observational samples.

Therefore, on large spatial scales, $j \leq J_{ss}$, the WFC covariance is already j -diagonal. Within this range, the covariance matrix is decomposed into j sub-matrices $\langle \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j,l'} \rangle$. This guide us to design the first statistics – the DWT band-power spectrum.

3.2. The DWT band-power spectrum

Because the model-predicted power spectrum is currently expressed in the Fourier representation, any statistical estimator designed for measuring the power spectrum from real data should have simple relation with the Fourier power spectrum.

Since we have only one realization of the cosmic mass field, no ensemble is available for each mode n . One cannot measure the Fourier power spectrum $P(n)$, as it is from the variance of the amplitude $|\delta_n|$ of mode n . Generally, a power spectrum estimator is to measure banded power spectrum as

$$P_j = \sum_n W_j(n) P(n), \quad (29)$$

where $W_j(n)$ is a window function, which is localized in the n (or Fourier) space. The problem that arises here is, what is the criterion for a reasonable banding? and how to optimize the banded power spectrum? The DWT representation provides a natural and reasonable way for the banding.

As discussed in §2.2, for an ergodic field, the 2^j WFCs $\tilde{\epsilon}_{j,l}$ at a given j formed an one point distribution of the fluctuations at the scale j . Therefore, the DWT power spectrum at the scale j can be defined as the variance of the one-point distribution, i.e.,

$$P_j = \frac{1}{2^j} \sum_{l=0}^{2^j-1} (\tilde{\epsilon}_{j,l} - \langle \tilde{\epsilon}_{j,l} \rangle)^2. \quad (30)$$

Because of the zero mean of WFC $\langle \tilde{\epsilon}_{j,l} \rangle$, [eq.(8)]. P_j can be written as, statistically,

$$P_j = \frac{1}{2^j} \sum_{l=0}^{2^j-1} |\tilde{\epsilon}_{j,l}|^2 = \frac{1}{2^j} \sum_{l=0}^{2^j-1} P_{j,l}, \quad (31)$$

which is an ergodicity-allowed spatial average of $P_{j,l}$, and is usually referred as DWT power spectrum. As we will show below, Eq.(31) gives an estimator of band-average Fourier power spectrum.

The DWT power spectrum eq.(31) is certainly less detailed than the power spectrum $P(n)$ or $P_{j,l}$. However, the numbers P_j are probably the maximum of statistically valuable band-power spectrum which can be extracted from one realization of an ergodic field. The optimum of this banding can be seen via the phase space $\{x, k\}$, where the wavenumber $k = 2\pi n/L$. Generally a set of orthogonal and complete basis of multiresolution analysis decomposes the entire phase space into elements with different shape, but their volume always satisfies the uncertainty relation, $\Delta x \cdot \Delta k \geq 2\pi$. The ordinary Fourier transform is not a multiresolution decomposition, but always takes highest resolution of k , i.e. $\Delta k \rightarrow 0$, and lowest resolution of x , $\Delta x \rightarrow \infty$.

To apply the ergodicity, we chopped the survey volume L into pieces Δx . If Δx is too large, or $L/\Delta x$ too small, the ensemble contains few members, and thus there will be larger

vertical errors placed on the estimated power spectrum. In order to minimize this error, we may make the size of chopped pieces Δx to be small. Correspondingly, the width of window function $\Delta k = 2\pi/\Delta x$ will broaden, and the scale resolution will be poor, i.e., there will be a large horizontal error bar placed on the estimated power spectrum. Thus, the optimal chopping can be achieved by a compromise between these two trade-off factors $L/\Delta x$ and Δk . Generally, $1/\Delta x$ is proportional to the resolvable wavenumber, i.e.

$$1/\Delta x \propto k. \quad (32)$$

therefore, the optimized banding $\Delta k \Delta x = 2\pi$ requires

$$\frac{\Delta k}{k} = \Delta \ln k \simeq 1. \quad (33)$$

That is, the optimized banding is in logarithmic spacing. To detect small scale fluctuations (larger wavenumber k), the size of the pieces Δx is chosen to be smaller. To detect large scale fluctuations (smaller wavenumber), the size of the pieces Δx is chosen to be larger. The wavelets $\psi_{j,l}(x)$ is constructed by dilating (i.e. changing scale) of the generating function by a factor 2^j (Appendix A). Therefore, we have $\Delta \ln k \sim 1$. In this sense, the DWT is an optimized multiscale decomposition (Farge 1992). Because the set of wavelet basis is complete, one cannot have more independent bands than P_j .

Under the assumption of a homogeneous Gaussian field, the DWT power spectrum eq.(31) can be rewritten as

$$P_j = \frac{1}{2^j} \sum_{n=-\infty}^{\infty} |\hat{\psi}(n/2^j)|^2 P(n). \quad (34)$$

where eqs.(11), (21) and (22) have been used. Comparing with eq.(29), clearly, P_j is a band-averaged Fourier power spectrum with the window function

$$W_j(n) = \frac{1}{2^j} |\hat{\psi}(n/2^j)|^2. \quad (35)$$

Generally, the function $\hat{\psi}(n)$ is non-zero in two narrow wavenumber ranges centered at $n = \pm n_p$ with width Δn_p . Therefore P_j is the band spectrum centered at

$$\ln n_j = j \log 2 + \log n_p, \quad (36)$$

with the band width as

$$\Delta \log n = \Delta n_p / n_p \quad (37)$$

which stays constant logarithmically. Eqs.(36) and (37) show that the countable data set $\{P_j, j = 1, 2, \dots\}$ represents scale-by-scale band-averaged Fourier power spectrum with the logarithmic spacing of wavenumber. P_j is completely determined by the Fourier power spectrum, and therefore, it should be effective for constraining the parameters contained in the Fourier power spectrum.

The band-power spectrum (31) can also be written as, alternatively,

$$P_j = \frac{1}{2^j} \text{tr } Cov_{l,l'}^j \quad (38)$$

where the matrix $Cov_{l,l'}^j$ is the j submatrix of the covariance, i.e.

$$Cov_{l,l'}^j = \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j,l'}. \quad (39)$$

Therefore, P_j 's exhaust all information of the j diagonals of the WFC covariance. Eq.(38) shows that we actually need not to diagonalize each j submatrix, as P_j is given by the trace of the j submatrix.

3.3. Scale-scale correlations in second and higher orders

In the range of $j > J_{ss}$, the scale-scale correlations become significant, the DWT covariance will no longer be diagonal or j -diagonal.

In this scale range, we should do somewhat diagonalization of the DWT covariance. However, the scale-scale correlation may lead to large errors of the diagonalization, even

the diagonalization becomes impossible. Let us consider the example of the scale-scale correlation given by eq.(25). In this case, the variable $\tilde{\epsilon}_{j+1;l}$ actually is linearly dependent on $\tilde{\epsilon}_{j,l+\Delta l}$, and therefore the matrix $\langle \tilde{\epsilon}_{j+1,l} \tilde{\epsilon}_{j,l'} \rangle$ is singular. It cannot be diagonalized. For instance, for scales $j = 1, 2$, the covariance matrix now is

$$\begin{pmatrix} \tilde{\epsilon}_{1,0}\tilde{\epsilon}_{1,0} & \tilde{\epsilon}_{1,0}\tilde{\epsilon}_{2,0} & \tilde{\epsilon}_{1,0}\tilde{\epsilon}_{2,1} \\ \tilde{\epsilon}_{2,0}\tilde{\epsilon}_{1,0} & \tilde{\epsilon}_{2,0}\tilde{\epsilon}_{2,0} & \tilde{\epsilon}_{2,0}\tilde{\epsilon}_{2,1} \\ \tilde{\epsilon}_{2,1}\tilde{\epsilon}_{1,0} & \tilde{\epsilon}_{2,1}\tilde{\epsilon}_{2,0} & \tilde{\epsilon}_{2,1}\tilde{\epsilon}_{2,1} \end{pmatrix} = \tilde{\epsilon}_{1,0}^2 \begin{pmatrix} 1 & a & b \\ a & a^2 & ab \\ b & ab & b^2 \end{pmatrix} \quad (40)$$

Obviously, this matrix cannot be diagonalized.

More seriously, if the matrix elements have some uncorrelated errors due to measurements, i.e. $\tilde{\epsilon}_{j,l}\tilde{\epsilon}_{j',l'} \pm \Delta\tilde{\epsilon}_{j,l,j',l'}$, the matrix (40) looks diagonalizable. However in this case the minors of the matrix are given by the errors $\Delta\tilde{\epsilon}_{j,l,j',l'}$, and therefore, the diagonalization will be largely contaminated by the errors.

This example indicates that when the scale-scale correlations appear, the number of the independent variables, and then the signal-to-noise ratio, will decrease. we should not extract the statistical properties of the covariance by a diagonalization.

Fortunately, our ultimate goal is not the mathematical diagonalization, but discrimination among physical models of the structure formation. An alternative to the full diagonalization is to take the following two measures: (1) Using the j -diagonals of each j to calculate the band-power spectrum P_j [eq.(31)]; (2) using the j off-diagonals to calculate the second order scale-scale correlations. The second order scale-scale correlations is defined as

$$\begin{aligned} C_{j,j'}(\Delta l) &= \frac{1}{2^j} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j',l'}, \quad j > j', \\ l' &= \text{mod}[l/2^{j-j'}] + \Delta l. \end{aligned} \quad (41)$$

Like the band-power spectrum [eqs.(30) and (31)], $C_{j,j'}(\Delta l)$ is defined by an ergodicity-allowed average. $C_{j,j'}(\Delta l)$ measures the second order correlation between fluctuations

on scale j and j' at positions l and l' . Since cosmic density field is homogeneous, the correlation depends only on the difference between l and l' , i.e. $\Delta l L / 2^{j'}$. For an initially Gaussian field, the scale-scale correlations are developed during the non-linear evolution of the gravitational clustering.

Now, we can use the two statistics P_j and $C_{j,j'}$ to discriminate among models. Actually, the two statistics discrimination would be more worth than the full diagonalization. For instance, the model-predicted galaxy power spectra on smaller scales are generally degenerate with respect to cosmological parameters, i.e. models with different cosmological parameters can yield the same galaxy power spectrum. This is because one always can choose the bias model parameters to fit the prediction with the observations. Therefore, to remove the degeneracy, an independent measure for constraining the bias models is necessary. The scale-scale correlation is found to be sensitive to the bias model (Feng, Deng & Fang 2000). Thus, for model discrimination, the j -diagonal power spectrum plus scale-scale correlation would be more useful than a full-diagonalization.

In a word, in the scale range of $j > J_{ss}$, we will extract the valid statistical information from the covariance by P_j and $C_{j,j'}(\Delta l)$.

It should be pointed out that even when all $C_{j,j'}(\Delta l)$ vanish, one cannot conclude that the system is scale-scale uncorrelated. In other words, that a decomposition X_i yields a diagonal covariance doesn't mean that the modes X_i are really statistical uncorrelated. There are many clustering models which have diagonal covariance, but mode-mode statistics are correlated on higher orders (Greiner, Lipa & Carruthers 1995.) A diagonal decomposition means only that mode-mode is uncorrelated on second order.

The higher order generalization of $C_{j,j'}(\Delta l)$ is straightforward. For instance one can

measure the fourth order scale-scale correlations by

$$C_{j,j'}^2(\Delta l) = \frac{1}{2^j} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l}^2 \tilde{\epsilon}_{j',l'}^2, \quad j > j', \quad (42)$$

$$l' = \text{mod}[l/2^{j-j'}] + \Delta l.$$

This correlation $C_{j,j'}^2(\Delta l = 0)$ is essentially the same as the so called band-band correlation defined by

$$T = \frac{\langle P_j P_{j+1} \rangle}{\langle P_j \rangle \langle P_{j+1} \rangle}. \quad (43)$$

It has been shown that the precision of the Fourier band-power spectrum estimator depends on the band-band correlation T (Meiksin & White 1998.) In the DWT representation, we arrive at the similar conclusion that when $C_{j,j'}(\Delta l)$ or $C_{j,j'}^2(\Delta l)$ are non-zero, i.e. when the DWT covariance is not j diagonal, we should test models by both the band-power spectrum and scale-scale correlations. For samples of large scale structure, the scale-scale correlations $C_{j,j'}^2(\Delta = 0)$ has been found to be significant on scales less about $10 h^{-1}$ Mpc (Pando et al 1998, Feng, Deng & Fang 2000.)

4. The DWT algorithm of data binning

In the following two sections, we will discuss the algorithm for estimating the band power spectrum P_j and scale-scale correlations $C_{j,j'}(\Delta l)$ from galaxy redshift surveys, and other samples of large scale structures.

If the position measurement is perfectly precise, the observed galaxy distribution can be written as

$$\rho^g(x) = \sum_{i=1}^{N_g} w_i \delta^D(x - x_i), \quad (44)$$

where N_g is the total number of galaxies, $\{x_i\}$ the position of the i -th galaxy, $0 \leq x_i \leq L$, w_i its weight, and δ^D is the Dirac- δ function. However, the position measurement has error

due to finite spatial resolution, and therefore, the distribution usually is somewhat given by a binned histogram.

The binning is performed by a convolution of the data with a binning function $W(x)$ as

$$\tilde{\rho}^g(x) = \Pi(x) \int W(x - x') \rho^g(x') dx' \quad (45)$$

in which $\Pi(x)$ is the sampling function defined as $\Pi(x) = \sum_l \delta^D(x - lL/2^j)$, where l labels the l -th bin. Obviously, the mesh-defined density distribution is given by $\tilde{\rho}^g(x) = \sum_l \rho_l^g \delta^D(x - lL/2^j)$, where $\rho_l^g = \int W(lL/2^j - x') \rho^g(x') dx'$ is a mass assignment at the l -th bin.

It is well known that the binning eq.(45) will result in spurious features of the Fourier power spectrum on scale around the Nyquist frequency of the FFT grid (e.g. Jing 1992, Percival & Walden 1993, Baugh & Efstathiou 1994). Mathematically, eq.(45) implies a decomposition by the weight function $W(x)$. In other word, $W(lL/2^j - x')$ are playing the role of a scaling functions (or sampling function.) If the scaling functions are orthogonal and complete, the one cannot recovered the original field without distortion. This may cause some spurious features, such as the aliasing effect in the FFT. In the DWT analysis, the binning or sampling are always done by an orthogonal and complete decomposition, one can expected that the spurious features and false correlations can be completely avoided.

4.1. Binning with wavelets

The WFCs $\tilde{\epsilon}_{j,l}$ are assigned at regular grids $l = 0 \dots 2^j - 1$. It is actually a binning of data. In this case, the binning is automatically realized by the orthogonal projection onto wavelet space, and no extra weight function is required. In result, the contamination due to the sampling error is naturally eliminated.

With eq.(6), one can directly calculate the WFCs of the galaxy distribution (44) by

$$\tilde{\epsilon}_{j,l}^g = \sum_{i=1}^{N_g} w_i \psi_{j,l}(x_i). \quad (46)$$

The errors of $\tilde{\epsilon}_{j,l}^g$ can also be calculated from the errors of x_i .

Since we used the periodized distribution $\delta(x)$ in eq.(6), the discontinuity between the data at two boundaries may introduce false coefficients. Yet, this possible false signal is only related to boundaries. One can expected that this false coefficients will not be important for detecting power spectrum on scales much less than L . This boundary effect has been tested numerically by using simulated samples over a finite length divided in 512 bins with two different boundary conditions (A) periodic boundary conditions; (B) zero padding. The results show that the spectrum can be correctly reconstructed by the DWT regardless of the boundary conditions on scales equal to and less than 64 bins (Pando & Fang 1998).

Note has to be taken of the difference between usual mass assignment and the DWT projection (46). In the former, the mass assignment is given by partitioning the mass on the grids according to the binning function $W(x)$, and the binning data are the mesh-defined densities. Whereas for the DWT projection, the binning data, i.e. the WFCs $\tilde{\epsilon}_{j,l}^g$ are not the mesh-defined densities, but the fluctuations on scale j at position l , which is obviously not positive-definite.

4.2. Binning with scaling functions

In the DWT analysis, the mass assignment is realized by the scaling function $\phi_{j,l}(x)$ [eq.(A30)]. Besides the orthogonality eqs.(A33) and (A34), the basic scaling function $\phi(\eta)$ (which is not yet periodized!) satisfies the so-called “partition of unity” as (Daubechies 1992)

$$\sum_{l=-\infty}^{\infty} \phi(\eta - l) = 1. \quad (47)$$

One can also define the periodized scaling function as

$$\phi_{j,l}^P(x) = \left(\frac{2^j}{L}\right)^{1/2} \sum_{n=-\infty}^{\infty} \phi[2^j(\frac{x}{L} + n) - l]. \quad (48)$$

Thus, eq.(47) can be rewritten as

$$\sum_{l=0}^{2^j-1} \frac{L}{2^j} \phi_{j,l}^P(x) = 1 \quad (49)$$

We will only use the periodized scaling function below, and drop the superscript P .

With the periodized scaling function, the eqs.(A39) - (A41) give

$$\rho(x) = \rho^J(x) + \sum_{j=J}^{\infty} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l} \psi_{j,l}(x), \quad (50)$$

where

$$\rho^J(x) = \sum_{l=0}^{2^J-1} \epsilon_{J,l} \phi_{J,l}(x). \quad (51)$$

The scaling function coefficients (SFCs) $\epsilon_{J,l}$ is given by

$$\epsilon_{J,l} = \int_0^L \rho(x) \phi_{J,l}(x) dx \quad (52)$$

Subjecting the distribution (44) to the transform eq.(50), we have

$$\rho^g(x) = \sum_{l=0}^{2^J-1} \epsilon_{J,l}^g \phi_{J,l}(x) + \sum_{j=J}^{\infty} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l}^g \psi_{j,l}(x), \quad (53)$$

where

$$\epsilon_{J,l}^g = \sum_{i=1}^{N_g} w_i \phi_{J,l}(x_i). \quad (54)$$

Using eqs.(44) and (54), eq.(49) yields

$$\sum_{l=0}^{2^j-1} \frac{L}{2^j} \epsilon_{j,l}^g = \sum_{i=1}^{N_g} w_i. \quad (55)$$

This shows that the i -th galaxy is assigned onto grid l by number $(L/2^j)w_i \phi_{J,l}(x_i)$.

Therefore, the SFC $(L/2^j)\epsilon_{j,l}^g$ is the mass assignment of $\rho^g(x)$.

4.3. The DWT binning and FFT

Given a galaxy distribution eq.(44), its Fourier transform is evaluated by the trigonometric summation

$$\hat{\rho}^g(n) = \sum_{i=1}^{N_g} w_i e^{i2\pi n x_i/L}, \quad (56)$$

and the power spectrum is $|\hat{\rho}^g(n)|^2$. However, the power spectrum given by the FFT of $\tilde{\rho}^g(x)$ [eq.(45)] is

$$|\hat{\rho}_l^g(n)|^2 = \sum_{n'=-\infty}^{\infty} |\hat{W}(n + 2^j n')|^2 |\hat{\rho}^g(n + 2^j n')|^2 \quad (57)$$

where $\hat{W}(n)$ is the FT of the binning function $W(x)$. The power spectrum (57) is obviously not equal to the power spectrum $|\hat{\rho}^g(n)|^2$. The power spectrum (57) is given by a superpositions of the power spectrum $|\hat{\rho}^g(n + 2^j n')|^2$ on all scales $n + 2^j n'$. This is the “aliasing” effect (Hockney & Eastwood 1989, Hoyle, et al. 1999).

In the DWT representation, the FT of eq.(53) yields

$$\hat{\rho}^g(n) = \sum_{l=0}^{2^J-1} \epsilon_{J,l}^g \hat{\phi}_{J,l}(n) + \sum_{j=J}^{\infty} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l}^g \hat{\psi}_{j,l}(n) \quad (58)$$

where the function $\hat{\phi}_{j,l}(n)$ is the Fourier transform of $\phi_{j,l}(x)$, i.e.

$$\hat{\phi}_{j,l}(n) = \int_{-\infty}^{\infty} \phi_{j,l}(x) e^{-i2\pi n x/L} dx. \quad (59)$$

Using the definition of $\phi_{j,l}(x)$ [eq.(A30)], eq.(59) becomes

$$\hat{\phi}_{j,l}(n) = \left(\frac{2^j}{L}\right)^{-1/2} \hat{\phi}(n/2^j) e^{-i2\pi n l/2^j} \quad (60)$$

where $\hat{\phi}(n)$ is the Fourier transform of the basic scaling function $\phi(\eta)$

$$\hat{\phi}(n) = \int_{-\infty}^{\infty} \phi(\eta) e^{-i2\pi n \eta} d\eta. \quad (61)$$

Eq.(58) gives then

$$\hat{\rho}^g(n) = \left(\frac{2^J}{L}\right)^{-1/2} \hat{\phi}(n/2^J) \sum_{l=0}^{2^J-1} \epsilon_{J,l}^g e^{-i2\pi n l/2^J} + \sum_{j=J}^{\infty} \left(\frac{2^j}{L}\right)^{-1/2} \hat{\phi}(n/2^j) \hat{\psi}(n/2^j) \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l}^g e^{-i2\pi n l/2^j}, \quad (62)$$

Since $\hat{\psi}(n/2^j)$ is localized in $n/2^j \sim n_p$, the second terms in the r.h.s. of eq.(62) are important only for $n \geq 2^J n_p$. Thus, the Fourier transform $\hat{\rho}^g(n)$ can be evaluated by

$$\hat{\rho}^g(n) = \hat{\phi}(n/2^J) \hat{F}(n/2^J), \quad n \leq 2^J n_p \quad (63)$$

where

$$\hat{F}(n/2^J) = \left(\frac{2^J}{L}\right)^{-1/2} \sum_{l=0}^{2^J-1} \epsilon_{J,l} e^{-i2\pi n l / 2^J}. \quad (64)$$

\hat{F} can be calculated by the standard FFT technique. Therefore, the FT of the galaxy distribution $\rho^g(x)$ can be evaluated directly by FFT of its SFC mass assignment $\epsilon_{J,l}^g$. Eqs.(63) and (64) is actually a scale-adaptive FFT for estimating the power spectrum of an irregular data set. This algorithm computes $\hat{\rho}^g(n)$ up to the scales $n \leq 2^J n_p$, where the adapted scale J can be chosen as high as the scales to be studied.

5. The DWT algorithm on the Poisson sampling

The observed or the mock galaxy distributions $\rho^g(x)$ are considered to be a Poisson sampling with an intensity $\rho^M(x) = \bar{\rho}(x)[1 + \delta(x)]$, where $\bar{\rho}(x)$ is the galaxy distribution if galaxy clustering is absent, and given by the selection function (Peebles 1980). A proper power spectrum estimator should be effective to obtain the power spectrum debiased from the Poisson sampling. It has been realized that, to handle the Poisson sampling with a non-uniform selection function, the decomposition basis $\psi_i(\mathbf{x})$ [eq.(1)] is required to have zero average (e.g. Tegmark et al. 1998), i.e.

$$\int \psi_i(x) dx = 0. \quad (65)$$

This is what we can take the advantage of the DWT analysis, as for the wavelets $\psi_{j,l}(x)$, eq.(65) always holds due to the admissibility [eq.(7)].

5.1. Algorithm for the DWT covariance affected by Poisson sampling

Considering the Poisson sampling, the characteristic function of the galaxy distribution $\rho^g(x)$ is

$$Z[e^i \int \rho^g(x) u(x) dx] = \exp \left\{ \int dx \rho^M(x) [e^{iu(x)} - 1] \right\}, \quad (66)$$

and the correlation functions of $\rho^g(x)$ are given by

$$\langle \rho^g(x_1) \dots \rho^g(x_n) \rangle_P = \frac{1}{i^n} \left[\frac{\delta^n Z}{\delta u(x_1) \dots \delta u(x_n)} \right]_{u=0}, \quad (67)$$

where $\langle \dots \rangle_P$ is the average for the Poisson sampling. We have then

$$\langle \rho^g(x) \rangle_P = \rho^M(x), \quad (68)$$

and

$$\langle \rho^g(x) \rho^g(x') \rangle_P = \rho^M(x) \rho^M(x') + \delta^D(x - x') \rho^M(x). \quad (69)$$

This equation yields

$$\langle \delta(\mathbf{x}) \delta(\mathbf{x}') \rangle = 1 + \left\langle \frac{\langle \rho^g(x) \rho^g(x') \rangle_P}{\bar{\rho}(x) \bar{\rho}(x')} \right\rangle - \delta^D(x - x') \frac{1}{\bar{\rho}(x)}. \quad (70)$$

Since $\bar{\rho}(x)$ is not subject to a Poisson process, the second term of the r.h.s. of eq.(70) can be rewritten as $\langle \langle [\rho^g(x)/\bar{\rho}(x)] [\rho^g(x')/\bar{\rho}(x')] \rangle_P \rangle$. Using eq. (44), we have

$$\frac{\rho^g(x)}{\bar{\rho}(x)} = \sum_{i=1}^{N_g} \frac{1}{\bar{\rho}(x_i)} w_i \delta^D(x - x_i). \quad (71)$$

in which the factor $\bar{\rho}(x_i)$ can be absorbed into the weight factors w_i . The WFC covariance is given by

$$\langle \tilde{\epsilon}_{j,l} \tilde{\epsilon}_{j',l'} \rangle = \langle \langle \tilde{\epsilon}_{j,l}^g \tilde{\epsilon}_{j',l'}^g \rangle_P \rangle - \int \frac{\psi_{j,l}(x) \psi_{j',l'}(x)}{\bar{\rho}(x)} dx. \quad (72)$$

The first term in r.h.s of eq.(70) disappears as all the basis functions $\psi_{j,l}(x)$ are admissible [eq.(7)].

5.2. The estimators for the DWT band power spectrums

If the selection function varies slowly on a scale j , i.e.

$$\frac{d \ln \bar{\rho}(x)}{dx} \ll 2^j / L, \quad (73)$$

we have approximately,

$$\int \frac{\psi_{j,l}(x) \psi_{j',l'}(x)}{\bar{\rho}(x)} dx = \frac{1}{\bar{\rho}(x_l)} \delta_{j,j'} \delta_{l,l'}, \quad (74)$$

where $\bar{\rho}(x_l)$ is the number density of galaxies averaged over a volume of $L/2^j$ at l . In this case, the band-power spectrum is simplified as

$$P_j = \frac{1}{2^j} \sum_{l=0}^{2^j-1} \langle \langle \tilde{\epsilon}_{j,l}^g \tilde{\epsilon}_{j,l}^g \rangle_P \rangle - \frac{1}{2^j} \sum_{l=0}^{2^j-1} \frac{1}{\bar{\rho}(x_l)}. \quad (75)$$

The second term in the r.h.s. is the variance from the Poisson process. Since the Poisson process does not change the ergodicity, the average over l in eq.(75) is already a fair estimation for the ensemble average. Therefore, one can drop $\langle \langle \dots \rangle_P \rangle$ in eq.(75), and the estimation of the DWT band power spectrum is given by

$$P_j = \frac{1}{2^j} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l}^g \tilde{\epsilon}_{j,l}^g - \frac{1}{2^j} \sum_{l=0}^{2^j-1} \frac{1}{\bar{\rho}(x_l)}. \quad (76)$$

The second term is for subtracting the contribution of the discreteness effect (or shot noise) in the Poisson sampling from the power spectrum. P_j is debiased from the Poisson process.

5.3. The estimators for the scale-scale corrections

Similarly, one can calculate the debiased scale-scale correlations from a galaxy sample $\rho^g(x)$. From eq.(70), the term of the Poisson process is free from scale-scale correlation, the second order scale-scale correlation can be calculated from the WFCs of the galaxy distribution without the correction for the shot noise

$$C_{j,j'}(\Delta l) = \frac{1}{2^j} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l}^g \tilde{\epsilon}_{j', \text{mod}[l/2^{j-j'}] + \Delta l}^g, \quad j > j'. \quad (77)$$

However, the Poisson process is not free from higher order scale-scale correlations. For instance, to estimate the band-band correlations eq.(42), we use eq.(67) with $n = 4$. It gives

$$C_{j,j'}^2 = \frac{1}{2^j} \left[\sum_{l=0}^{2^j-1} (\tilde{\epsilon}_{j,l}^g)^2 (\tilde{\epsilon}_{j',l'}^g)^2 \right. \\ - 2 \sum_{l=0}^{2^j-1} \int \frac{\psi_{j,l}(x) \psi_{j',l'}(x)}{\bar{\rho}(x)} dx \int \frac{\psi_{j,l}(x') \psi_{j',l'}(x')}{\bar{\rho}(x')} dx' \\ \left. - \sum_{l=0}^{2^j-1} \int \frac{\psi_{j,l}^2(x)}{\bar{\rho}(x)} dx \int \frac{\psi_{j',l'}^2(x')}{\bar{\rho}(x')} dx - \sum_{l=0}^{2^j-1} \int \frac{\psi_{j,l}^2(x) \psi_{j',l'}^2(x)}{\bar{\rho}^3(x)} dx \right]. \quad (78)$$

where $j > j'$ and $l' = \text{mod}[l/2^{j-j'}] + \Delta l$. The last three terms are the scale-scale correlations $C_{j,j'}^2$ from the Poisson sampling. Exactly, the factor $\bar{\rho}(x)$ in the Poisson terms should be $\rho^M(x) = \bar{\rho}(x)[1 + \delta(x)]$, but we ignored the contributions of $\delta(x)$ at the moment.

If the selection function is slowly varying on scales j and j' [eq.(73)], we have

$$C_{j,j'}^2 = \frac{1}{2^j} \left[\sum_{l=0}^{2^j-1} (\tilde{\epsilon}_{j,l}^g)^2 (\tilde{\epsilon}_{j',l'}^g)^2 \right. \\ \left. - \sum_{l=0}^{2^j-1} \frac{1}{\bar{\rho}(x_l) \bar{\rho}(x_{l'})} - \sum_{l=0}^{2^j-1} \int \frac{\psi_{j,l}^2(x) \psi_{j',l'}^2(x)}{\bar{\rho}^3(x)} dx \right]. \quad (79)$$

The second and third terms correct for the shot noise on the 4-th order. Numerical results showed that for typical samples of galaxy survey the local ($l' = l$) scale-scale correlation of the Poisson sampling is significant on small scales (Feng, Deng & Fang 2000.)

6. Discussions and conclusions

We presented the method of extracting the band-power spectrum from observed data and simulation sample via a DWT multiresolution decomposition. The DWT scale-by-scale approach provides a physical insight into the covariance matrix of the cosmic mass field.

A key indicator of the DWT power spectrum estimator is the scale-scale and/or the band-band correlations, which can be calculated directly from the DWT covariance and

the WFCs. In the scale range that the scale-scale correlations are negligible, the DWT covariance is $j(\text{scale})$ -diagonal, and it is already a lossless estimation of a banded power spectrum P_j . This DWT band power spectrum is optimized in the sense that the spatial resolution is adaptive automatically to the scales of the density perturbations.

In the scale range that the scale-scale (or band-band) correlations are significant, the diagonalization of the covariance may not yield an accurate power spectrum, but seriously contaminated by errors. In this case, an effective confrontation between the observed sample and model-prediction may not be given by a full diagonalized covariance, but both of the DWT power spectrum and scale-scale correlations. With the DWT representation, one can calculate the scale-scale correlation as well as the DWT power spectrum. Therefore, the DWT covariance is also useful when scale-scale correlation is strong.

In summary, the basic DWT algorithm is proceeded in the following steps,

1. Calculation of the WFCs $\tilde{\epsilon}_{J,l}^g$ and/or the SFCs $\epsilon_{J,l}^g$ from the data $\rho^g(x)$, where J corresponds to the highest resolution of the samples.
2. Calculation of the WFCs $\tilde{\epsilon}_{j,l}^g$ for various scale j .
3. Calculate the band-power spectrum P_j , and scale-scale correlations $C_{j,j'}$.
4. In the j range of $C_{j,j'} \simeq 0$, testing models or constraining parameters by comparing the model-predicted DWT band-power spectrum P_j with observed results.
5. In the j range of $C_{j,j'} \neq 0$, testing model or constraining parameters by comparing the model-predicted DWT band-power spectrum and scale-scale correlations with observed results.

Since the DWT is computationally powerful, the above-mentioned algorithm is found to be numerically efficient and flexiable (Yang et al. 2000.) Moreover, the developed

method is open in the sense that based on the WFCs and SFCs one can add subsequent items to realize the further goals related to the power spectrum measurement and model discrimination. Some of these problems are discussed below.

6.1. Higher dimensions and complex geometry

The DWT analysis in a 2 and/or 3-D space \mathbf{x} can be performed by the bases of the 1-D bases direct product, i.e.

$$\psi_{(j_1, j_2, j_3), (l_1, l_2, l_3)}(x_1, x_2, x_3) = \psi_{j_1, l_1}(x_1) \psi_{j_2, l_2}(x_2) \psi_{j_3, l_3}(x_3). \quad (80)$$

In this case, the three scales (j_1, j_2, j_3) of the WFCs can be different for different directions. One can define radial scales by

$$k = 2\pi \left[\left(\frac{2^{j_1}}{L_1} \right)^2 + \left(\frac{2^{j_2}}{L_2} \right)^2 + \left(\frac{2^{j_3}}{L_3} \right)^2 \right]^{1/2}, \quad (81)$$

where $L_1 \times L_2 \times L_3$ is the 3-D box.

For 2 and 3-D samples, one can also decompose by the mixed direct product of 1-D wavelets and scaling functions. For instance, a 3-D sample can be decomposed by bases

$$\psi_{(j_1, j_2, j_3), (l_1, l_2, l_3)}^{(1,2)}(x_1, x_2, x_3) = \phi_{j_1, l_1}(x_1) \psi_{j_2, l_2}(x_2) \psi_{j_3, l_3}(x_3). \quad (82)$$

where the scaling functions $\phi_{j,l}$ actually play the role of chopping a 3-D sample into 2^{j_1} 2-D slices in the x_1 direction, $l_1 = 0, \dots, 2^{j_1} - 1$. Like the binning by the scaling function (§4.2), the chopping eq.(82) will not cause spurious features.

The problem of complex geometry of samples can be treated by using the locality of the $\psi_{j,l}$ (Pando & Fang 1998a). The locality property allows the WFCs to be independent of the data outside an “influence” cone. The WFCs $\tilde{\epsilon}_{j,l}$ is only determined by data in the interval $[(lL/2^{j+1} - (\Delta x)/2^{j+1}, (lL/2^{j+1} + (\Delta x)/2^{j+1})]$, where Δx is the width of the basic

wavelet ψ . With this property, any complex geometry of samples can be regularized into a 2 or 3-D box by zero padding in the field between the sample geometry and the box. Since all WFCs at the zero padding zone are zero, one can use the DWT to analyze the regular box, but not treat the WFCs related to the zero padding as the variables of valid degrees of freedom.⁴

6.2. Non-Gaussianity and power spectrum detection

We have emphasized that the information of the non-Gaussian features are important for a precise detection of the power spectrum, or band power spectrum. That is because, from the covariance, one can only find statistically uncorrelated (or statistical orthogonal) bases or modes on second order. For non-Gaussian fields, the modes statistically uncorrelated on second order might be statistically correlated at the 3rd and 4th orders. On the other hand, the power spectrum is of second order, and therefore, the power spectrum estimates at different scales might not be statistically uncorrelated if there are 3rd and 4th order correlations. The accuracy of a power spectrum estimation is affected by the higher order statistical correlations.

For instance, a popular bias model for galaxy formation employ the selection probability functions as (Cole et al. 1998)

$$P(\delta(\mathbf{r})) \propto \exp \left[\alpha \frac{\delta_s(\mathbf{r})}{\sigma_s} \right], \quad (83)$$

where α is const, and $\delta_s(\mathbf{r})$ and σ_s are smoothed density field and variance. Therefore, if the density field is Gaussian, the galaxy distribution given by the Poisson sampling with the

⁴About DWT on manifold, see also W. Sweldens <http://cm.bell-labs.com/who/wim> or <http://www.wavelet.org>

intensity eq.(83) will be lognormal. The baryonic distribution is sometimes also modeled by a lognormal relation with the underlying Gaussian mass field (Bi, Ge & Fang 1995, Bi & Davidsen 1997). As having been well known, for lognormal distribution, the most likely value can be significantly different from their mean value. In this case, to estimate the accuracy of a power spectrum detection, the higher order cumulant statistics is needed.

In the DWT analysis, the 2^j WFCs give the one point distribution of the fluctuations on scale j . Therefore, the third and forth cumulants can be calculated by

$$S_j = \frac{1}{P_j^{3/2}} \frac{1}{2^j} \sum_{l=0}^{2^j-1} (\tilde{\epsilon}_{j,l} - \overline{\tilde{\epsilon}_{j,l}})^3, \quad (84)$$

$$K_j \equiv \frac{1}{P_j^2} \frac{1}{2^j} \sum_{l=0}^{2^j-1} (\tilde{\epsilon}_{j,l} - \overline{\tilde{\epsilon}_{j,l}})^4 - 3 \quad (85)$$

These are, respectively, the skewness and kurtosis spectra. It is not difficult to generalize eqs.(84) and (85) to more higher orders.

6.3. Selection of the basis of the multiresolution analysis

In computing the samples of redshift surveys, there are two coordinate systems having been widely used: 1. parallel plane system; 2. spherical shell system. For system 1, the volume of the survey can be approximated as a box, and therefore, the wavelets of eqs.(80) and (82) are suitable for the decomposition. For the system 2, we should use the wavelets on 2-D spherical surface. With the development of the DWT analysis, the bank of the DWT analysis has stored more and more sets of the orthogonal and complete basis for the multiresolution decomposition of different geometries. The multiscale analysis on geometry beyond above-mention two simple cases is being feasible.

6.4. Systematic effects

The influence of various systematic effects on the power spectrum detection has only been studied very preliminarily. The linear effect of redshift distortion on the power spectrum detection has been well studied (e.g. Hamilton 1995). It is not difficult to incorporate the linear theory of the redshift distortion with the DWT analysis. A key operator of the mapping a real space distribution into redshift space is $(1 - a(\partial^2/\partial z^2)\nabla^{-2})$, where coefficient a is const. To diagonalize this differential-integral operator, the Fourier representation is certainly the best. However, it has been shown that this operator is quasidiagonal in the DWT representation (Farge 1996).

Moreover, it would be straightforward to include a scale-dependent bias in the DWT representation. The redshift distortion is usually calculated under the assumption that the galaxy distribution $\rho^g(\mathbf{x})$ is linearly related to the underlying mass field $\rho(\mathbf{x})$, i.e. $\rho^g(\mathbf{r}) = b\rho(\mathbf{r})$, where b is the bias parameter. However, observations have indicated that the bias parameters probably are scale-dependent (Fang, Deng & Xia 1998.) It is easy to introduce scale-dependent bias in the DWT representation. For instance one can define a bias parameter on scale by $\tilde{\epsilon}_{j,l}^g = b_j\tilde{\epsilon}_{j,l}$.

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A. The discrete wavelet transform (DWT) of density fields

Let us briefly introduce the DWT analysis of the cosmic mass density fields, for the details of mathematical stuffs refers to the classical papers by Mallat (1989a,b,c); Meyer (1992); Daubechies, (1992) and references therein, and for physical applications, refers to Fang & Thews (1998) and references therein. Some other cosmological applications of wavelets can also be found at, e.g., Pando, Vills-Gabaud & Fang (1998), Hobson, Jones & Lasenby (1999), Sanz et al. (1999), Tenorio et al. (1999), Xu, Fang, & Wu (2000), Cayon, et al (2000).

A.1. Expansion by scaling functions

We consider here a 1-D mass density distribution $\rho(x)$ or contrast $\delta(x) = [\rho(x) - \bar{\rho}]/\bar{\rho}$, which are mathematically random fields over a spatial range $0 \leq x \leq L$. It is not difficult to extend all results developed in this section into 2-D and 3-D because the DWT bases for higher dimension can be constructed by a direct product of 1-D bases.

First, we introduce the scaling functions for the Haar wavelets. There are top-hat window functions defined by

$$\phi_{j,l}^H(x) = \begin{cases} 1 & \text{for } Ll2^{-j} \leq x \leq L(l+1)2^{-j} \\ 0 & \text{otherwise.} \end{cases}, \quad (\text{A1})$$

where the superscript H is stand for Haar. The scaling function, $\phi_{j,l}^H(x)$ actually gives a window at resolution scale $L/2^j$ and position $Ll2^{-j} \leq x \leq L(l+1)2^{-j}$. With the scaling function, the mean of density contrast distribution in the spatial range $Ll2^{-j} \leq x \leq L(l+1)2^{-j}$ can be expressed as

$$\epsilon_{j,l} = \frac{2^j}{L} \int_0^L \delta(x) \phi_{j,l}^H(x) dx. \quad (\text{A2})$$

The number $\epsilon_{j,l}$ is called the scaling function coefficient(SFC). Using SFCs, one can construct a density contrast field as

$$\delta^j(x) = \sum_{l=0}^{2^j-1} \epsilon_{j,l} \phi_{j,l}^H(x). \quad (\text{A3})$$

This is the density contrast $\delta(x)$ smoothed on scale $L/2^j$, or for simple, j -scale.

The scaling function $\phi_{j,l}^H(x)$ can be rewritten

$$\phi_{j,l}^H(x) = \phi^H(2^j x/L - l), \quad (\text{A4})$$

where

$$\phi^H(\eta) = \begin{cases} 1 & \text{for } 0 \leq \eta \leq 1 \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A5})$$

j, l are integers, with $j \geq 0$, and $0 \leq l \leq 2^j - 1$. $\phi^H(\eta)$ is called the basic scaling function.

The scaling function $\phi_{j,l}^H(x)$ is thus a translation and dilation of the basic scaling function.

The functions $\phi_{j,l}^H(x)$ are orthogonal with respect to l , i.e.

$$\int_0^L \phi_{j,l}^H(x) \phi_{j,l'}^H(x) dx = \frac{L}{2^j} \delta_{l,l'} \quad (\text{A6})$$

where $\delta_{l,l'}$ is Kronecker delta function. Thus, eq.(A3) gives functions in the function space V_j spanned by bases $\phi_{j,l}^H(x)$. V_j is a closed subspaces of $L_2(R)$, i.e. $V_j \subset L_2(R)$. It is easy to show that

$$\phi_{j,l}^H(x) = \phi_{j+1,2l}^H(x) + \phi_{j+1,2l+1}^H(x) \quad (\text{A7})$$

$$\epsilon_{j,l} = \frac{1}{2}(\epsilon_{j+1,2l} + \epsilon_{j+1,2l+1}). \quad (\text{A8})$$

Therefore, $V_j \subset V_{j+1}$ for all j . Thus, the orthogonal projectors P_j onto V_j , i.e. $P_j f \in V_j$, satisfy

$$\lim_{j \rightarrow \infty} P_j f = f, \quad (\text{A9})$$

for all $f \in L_2(R)$. A multiresolution analysis is then defined by the sequence of subspaces V_j .

A.2. Expansion by wavelets

Eqs. (A7) and (A8) show that $\delta^j(x)$ contains less information than $\delta^{j+1}(x)$, because information on scale $j+1$ have been smoothed out by eq. (A8). It would be nice not to lose any information during the smoothing from $j+1$ to j [eq.(A8)]. This can be accomplished if the differences, $\delta^{j+1}(x) - \delta^j(x)$, between the smoothed distributions on succeeding scales are somehow retained. This is, if we are able to retain these *differences*, this scheme will then make it possible to smooth the distribution and yet not lose any information as a result of the smoothing.

To calculate the *differences*, we define the difference function, or wavelet, as

$$\psi^H(\eta) = \begin{cases} 1 & \text{for } 0 \leq \eta \leq 1/2 \\ -1 & \text{for } 1/2 \leq \eta \leq 1 \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A10})$$

This is the basic Haar wavelet. As with the scaling functions, one can construct a set of wavelets $\psi_{j,l}^H(x)$ by dilating and translating eq.(A10) as

$$\psi_{j,l}^H(x) = \psi^H(2^j x/L - l). \quad (\text{A11})$$

The Haar wavelets are orthogonal with respect to *both* indexes j and l , i.e.

$$\int_0^L \psi_{j',l'}^H(x) \psi_{j,l}^H(x) dx = \left(\frac{L}{2^j}\right) \delta_{j',j} \delta_{l',l}. \quad (\text{A12})$$

For a given j , $\psi_{j,l}^H(x)$ is also orthogonal to the scaling functions $\phi_{j',l}^H(x)$ with $j' \leq j$, i.e.

$$\int_0^L \phi_{j',l'}^H(x) \psi_{j,l}^H(x) dx = 0, \quad \text{if } j' \leq j. \quad (\text{A13})$$

From eqs.(A4) and (A11), we have

$$\phi_{j,2l}^H(x) = \frac{1}{2}(\phi_{j-1,l}^H(x) + \psi_{j-1,l}^H(x)), \quad (\text{A14})$$

$$\phi_{j,2l+1}^H(x) = \frac{1}{2}(\phi_{j-1,l}^H(x) - \psi_{j-1,l}^H(x)).$$

Thus, the difference $\delta^{j+1}(x) - \delta^j(x)$ is given by

$$\delta^{j+1}(x) - \delta^j(x) = \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l} \psi_{j-1,l}^H(x), \quad (\text{A15})$$

where $\tilde{\epsilon}_{j,l}$ are called the wavelet function coefficients(WFC), which is given by

$$\tilde{\epsilon}_{j,l} = \frac{2^j}{L} \int \delta(x) \psi_{j,l}^H(x) dx. \quad (\text{A16})$$

Using the relation (A15) repeatedly, we have

$$\delta^j(x) = \delta^0(x) + \sum_{j'=0}^{j-1} \sum_{l=0}^{2^{j'}-1} \tilde{\epsilon}_{j',l} \psi_{j',l}^H(x). \quad (\text{A17})$$

This is an expansion of the function $\delta^j(x)$ with respect to the basis $\psi_{j,l}^H(x)$, and $\delta^0(x)$ is the mean of $\delta(x)$ in the range L . We have $\delta^0(x) = 0$ if $\delta(x)$ is density contrast. Considering (A9), for any $f(x) \in L^2(R)$ in L with mean $\bar{f} = 0$ we have

$$f(x) = \sum_{j=0}^{\infty} \sum_{l=0}^{2^j-1} \tilde{\epsilon}_{j,l} \psi_{j,l}^H(x), \quad (\text{A18})$$

and

$$\tilde{\epsilon}_{j,l} = \frac{2^j}{L} \int_0^L f(x) \psi_{j,l}^H(x) dx. \quad (\text{A19})$$

For a given j , the wavelets $\psi_{j,l}^H(x)$ form a space W_j which is the orthogonal complements of V_j in V_{j+1} , i.e. $V_{j+1} = V_j \oplus W_j$. Thus, every $f^j \in V_j$ has a unique decomposition $f^j = f^{j-1} + d^{j-1}$ with $f^{j-1} \in V_{j-1}$ and $d^{j-1} \in W_{j-1}$. Since $W_j \subset V_{j+1}$ and W_j is orthogonal to V_j , W_j is also orthogonal to W_{j-1} and W_{j+1} . Thus, all the spaces W_j are mutually orthogonal. Since V_j contains only $W_{j'}$ with $j' < j$, V_j is orthogonal to all $W_{j'}$ with $j' \geq j$.

A.3. Compactly supported orthogonal basis

In terms of the subspace V_j , the basic scaling function $\phi(\eta)$ and basic $\psi(\eta)$ belong to V_0 and W_0 respectively, and they can be expressed by the basis of V_1 , $\phi(2\eta - l)$, i.e.

$$\phi(\eta) = \sum_{l=-\infty}^{\infty} a_l \phi(2\eta - l), \quad (\text{A20})$$

$$\psi(\eta) = \sum_{l=-\infty}^{\infty} b_l \phi(2\eta - l), \quad (\text{A21})$$

where a_l and b_l are called the filter coefficients.

If we require that the scaling function $\phi(\eta)$ is normalized, eq.(A21) yields

$$\sum_l a_l = 2. \quad (\text{A22})$$

Requiring orthogonality for $\phi(x)$ with respect to discrete integer translations, i.e.

$$\int_{-\infty}^{\infty} \phi(\eta - m) \phi(\eta) d\eta = \delta_{m,0}, \quad (\text{A23})$$

we have

$$\sum_l a_l a_{l+2m} = 2\delta_{0,m}. \quad (\text{A24})$$

The orthogonality between ϕ and ψ means

$$\int_{-\infty}^{\infty} \psi(\eta) \phi(\eta - l) d\eta = 0. \quad (\text{A25})$$

Therefore, one has

$$b_l = (-1)^l a_{1-l}. \quad (\text{A26})$$

Furthermore, the wavelet $\psi(\eta)$ has to be admissible

$$\int_{-\infty}^{+\infty} \psi(\eta) d\eta = 0, \quad (\text{A27})$$

so we need

$$\sum_l b_l = 0. \quad (\text{A28})$$

The conditions (A22), (A24), (A26) and (A28) for the filter coefficients were employed to construct families of scaling functions and wavelets. The simplest solution of the filter coefficients is $a_0 = a_1 = b_0 = -b_1 = 1$ and all others 0. This solution gives the Haar wavelet. After the Haar wavelet, the simplest solution for the filter coefficients is

$$\begin{aligned} a_0 &= (1 + \sqrt{3})/4, & a_1 &= (3 + \sqrt{3})/4, \\ a_2 &= (3 - \sqrt{3})/4, & a_3 &= (1 - \sqrt{3})/4. \end{aligned} \quad (\text{A29})$$

This is the Daubechies 4 wavelet (D4). It is compactly supported and continuous.

With these wavelets, the multiresolution analysis can be performed in the similar way as developed in last two sections for the Haar wavelets. The scaling functions and wavelets for spanning the subspace V_j and W_j are given, respectively, by a translation and dilation of the basic scaling function and basic wavelet

$$\phi_{j,l}(x) = \left(\frac{2^j}{L}\right)^{1/2} \phi(2^j x/L - l) \quad (\text{A30})$$

and

$$\psi_{j,l}(x) = \left(\frac{2^j}{L}\right)^{1/2} \psi(2^j x/L - l). \quad (\text{A31})$$

The wavelets are orthonormal, i.e.

$$\int \psi_{j,l}(x) \psi_{j',l'}(x) dx = \delta_{j,j'} \delta_{l,l'}. \quad (\text{A32})$$

Eqs.(A23) and (A25) yield also

$$\int \phi_{j,l}(x) \phi_{j',l'}(x) dx = \delta_{l,l'}, \quad (\text{A33})$$

and

$$\int \phi_{j,l}(x) \psi_{j',l'}(x) dx = 0 \quad j' \geq j. \quad (\text{A34})$$

The set of $\psi_{j,l}$ and $\phi_{0,m}(x)$ with $0 \leq j < \infty$ and $-\infty < l, m < \infty$ form a complete, orthonormal basis in the space of functions with period length L .

Thus, a density field $\rho(x)$ with period length L can be expanded as (Fang & Thews 1998)

$$\rho(x) = \bar{\rho} + \bar{\rho} \sum_{j=0}^{\infty} \sum_{l=-\infty}^{\infty} \tilde{\epsilon}_{j,l} \psi_{j,l}(x), \quad (\text{A35})$$

or the density contrast $\delta(x) = (\rho(x) - \bar{\rho})/\bar{\rho}$ is

$$\delta(x) = \sum_{j=0}^{\infty} \sum_{l=-\infty}^{\infty} \tilde{\epsilon}_{j,l} \psi_{j,l}(x), \quad (\text{A36})$$

where

$$\bar{\rho} = L^{-1} \int_0^L \rho(x) dx \quad (\text{A37})$$

and

$$\tilde{\epsilon}_{j,l} = \int_{-\infty}^{\infty} \delta(x) \psi_{j,l}(x) dx. \quad (\text{A38})$$

More generally, we have

$$\rho(x) = \rho^J(x) + \bar{\rho} \sum_{j=J}^{\infty} \sum_{l=-\infty}^{+\infty} \tilde{\epsilon}_{j,l} \psi_{j,l}(x), \quad (\text{A39})$$

where $\rho^J(x)$ is the density field smoothed on scale J

$$\rho^J(x) = \sum_{l=-\infty}^{+\infty} \epsilon_{J,l} \phi_{J,l}(x). \quad (\text{A40})$$

and the scaling function coefficient(SFC) $\epsilon_{J,l}$ is given by

$$\epsilon_{J,l} = \int_{-\infty}^{+\infty} \rho(x) \phi_{J,l}(x) dx. \quad (\text{A41})$$

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